$$\int \tan x \, dx$$

$$\int \underbrace{2xe^{x^2}} dx = e^{x^2} + C$$

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

<u>Rewrite this:</u>

if F is an antiderivative of f, then

$$\frac{d}{dx}\left[F(g(x))\right] = F'(g(x))g'(x)$$

$$\frac{d}{dx}\left[F(g(x))\right] = f(g(x))g'(x)$$

integrate with respect to x

$$\cancel{\int} \cancel{\frac{d}{dx}}\left[F(g(x))\right] \cancel{dx} = \int f(g(x))g'(x)\,dx$$

$$\boxed{\int \underbrace{f(g(x))}\,\underbrace{g'(x)}\,dx = F(g(x)) + C}$$

e^{2x}

$\shortparallel$

$(e^x)^2$

$$\int \underbrace{2x}\,\underbrace{e^{x^2}}\,dx$$

$$= \underbrace{e^{x^2}}_{F(g(x))} + C$$

$\underbrace{f(g(x)) = e^{x^2}}$

$f(u) = e^u$

$F(u) = e^u$

$u = g(x) = x^2$

$\underline{g'(x) = 2x}$

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u-Substitution

Let $u = g(x)$, then

$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$$

$$\left(u = g(x) \longrightarrow \frac{du}{dx} = g'(x) \longrightarrow du = g'(x)\,dx\right)$$

Ex: ① $\int \underbrace{x^2}\,\underbrace{\sqrt{x^3+1}}\,\underbrace{dx}$

$$\left(\begin{array}{l} u = x^3+1 \\ du = 3\underbrace{x^2\,dx} \\ \hookrightarrow \frac{1}{3}du = \underline{x^2\,dx} \end{array} \right)$$

$$= \int \sqrt{u}\ \frac{1}{3}\,du$$

$$= \frac{1}{3}\int u^{1/2}\,du = \frac{1}{3}\left(\frac{2}{3}u^{3/2} + C\right)$$

$$= \boxed{\frac{2}{9}(x^3+1)^{3/2} + C}$$

② $\int (5-3x)^{10}\,dx$

$$\left(\begin{array}{l} u = 5-3x \\ du = -3\,dx \to -\frac{1}{3}du = dx \end{array} \right)$$

$$= \int u^{10}\left(-\frac{1}{3}du\right) = -\frac{1}{3}\int u^{10}\,du = -\frac{1}{3}\left(\frac{1}{11}u^{11} + C\right)$$

$$= \boxed{-\frac{1}{33}(5-3x)^{11} + C}$$

$f(3) = \frac{1}{3}$ $\quad$ $f(5) = \frac{1}{5}$

③ $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = \int \underline{\sin x}\,\underbrace{\frac{1}{\cos x}}\,\underline{dx}$

$$\left. \begin{array}{l} u = \cos x \\ du = -\sin x\,dx \end{array} \right)$$

$$= \int \frac{1}{u}(-du) = -\int \frac{1}{u}\,du$$

$$\begin{pmatrix} -\ln|\cos x| \\ = \ln\left(|\cos x|^{-1}\right) \\ = \ln|\sec x| \end{pmatrix} = -\ln|u| + C = \boxed{-\ln|\cos x| + C}$$

$$= \ln|\sec x| + C$$

(4) $\displaystyle\int \frac{4x^2}{x^3+1}\,dx$

$u = x^3 + 1$

$du = 3x^2\,dx \longrightarrow dx = \dfrac{du}{3x^2}$

$$= \int \frac{4\cancel{x^2}}{u}\,\frac{du}{3\cancel{x^2}} = \int \frac{4}{3u}\,du = \frac{4}{3}\int \frac{1}{u}\,du$$

$$= \frac{4}{3}\ln|u| + C = \frac{4}{3}\ln|x^3+1| + C$$

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(5) $\displaystyle\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx$

$$\begin{pmatrix} u = \frac{1}{x} = x^{-1} \\ du = -x^{-2}\,dx = \frac{-1}{x^2}\,dx \\ \hookrightarrow dx = -x^2\,du \end{pmatrix}$$

$$= \int \frac{\sin(u)}{\cancel{x^2}}\left(-\cancel{x^2}\,du\right)$$

$$= \int -\sin u\,du = \cos u + C = \cos\left(\frac{1}{x}\right) + C$$